CALCULUS.

395. Proposed by W. W. BURTON, Mercer University, Macon, Ga.

Into a full conical wine glass whose depth is a and whose angle at the base is 2α there is carefully dropped a spherical ball of such size as to cause the greatest overflow. Show that the radius of the ball is $a \sin \alpha/(\sin \alpha + \cos 2\alpha)$.

From Woods and Bailey's *A Course in Mathematics* (1907), Volume I, page 213.

396. Proposed by ELBERT H. CLARKE, Purdue University.

The length of the curve $y = x^n$ from the origin to the point (1, 1) is given by the formula

$$l = \int_0^1 \sqrt{1 + n^2x^{2n-2}}dx.$$

Our geometric intuition would tell us that the limit of this length as n becomes infinite is 2. Give a strict analytic proof that

$$\lim_{n\to\infty} \int_0^1 \sqrt{1 + n^2x^{2n-2}}dx = 2.$$

MECHANICS.

315. Proposed by H. S. UHLER, Yale University.

A solid, homogeneous, right, circular cylinder is allowed to move from rest down a circular cylindrical track which is concave upwards. Find the ratio of the radius of the track to the radius of the cylinder when the time of descent through a finite arc to the bottom is the same for the extreme cases of no slipping and zero friction. Show also that the same relation holds for a sphere descending a cylindrical or spherical surface.

316. Proposed by C. N. SCHMALL, New York, N. Y.

A body at rest at a point R begins to move towards a center of force F. The distance $RF = d$, and the force varies inversely as the distance. Two intermediate points in the path are P and Q, such that $FP = kd$, and $FQ = k^nd$. Show that the body will traverse the distance QP in a *maximum* of time if $k = 1/n^{\frac{1}{2(n-1)}}$.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

426. Proposed by HERBERT N. CARLETON, West Newbury, Mass.

Find all solutions of the equation

$$x\sqrt[x]{x} = x^x.$$

Solution by J A. Capron, Notre Dame, Ind.

The equation may be written in the form $x^{1+(1/x)} = x^x$, or $x^{(x+1)/x} - x^x = 0$. Factoring, we have $x^x[x^{(x+1-x^2)/x} - 1] = 0$. This equation is equivalent to the two equations $x^x = 0$ and $x^{(x+1-x^2)/x} - 1 = 0$. The first of these equations is satisfied for the value of $x = -\infty$. From the second equation, we have, by taking logarithms, the equation

$$\left(\frac{x + 1 - x^2}{x}\right) \log x = 0.$$

This equation is equivalent to the three equations $1/x = 0$, $x + 1 - x^2 = 0$, and $\log x = 0$. From the first of these equations, $x = \pm\infty$; from the second, $x = (1 \pm \sqrt{5})/2$; and from the third, $x = 1$.

By substituting the values of x found above in the original equation, we see that 0 and $\pm\infty$ are to be rejected. We find, however, by inspection that $x = 1$ is a root.

Hence, the roots are 1 and $(1 \pm \sqrt{5})/2$.

Also solved by ALBERT N. NAUER, A. M. HARDING, C. E. GITHENS, V. M. SPUNAR, ELIJAH SWIFT, W. C. EELLS, G. W. HARTWELL, and the PROPOSER.

430A. Proposed by H. C. FEEMSTER, York College, Neb.

Solve the equations

$$\sum_{i=1}^{n} x_i - x_n = k + \frac{n^2 - 3n + 2}{2} d, \tag{1}$$

$$\sum_{i=1}^{n} x_i - x_{n-1} = k + \frac{n^2 - 3n + 4}{2} d, \tag{2}$$

$$\sum_{i=1}^{n} x_i - x_{n-2} = k + \frac{n^2 - 3n + 6}{2} d, \tag{3}$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$\sum_{i=1}^{n} x_i - x_1 = k + \frac{n^2 - n}{2} d. \tag{n}$$

SOLUTION BY A. M. HARDING, Univ. of Arkansas.

Add the given equations and obtain

$$(n-1)\sum_{i=1}^{n} x_i = nk + \frac{n^3 - 3n^2 + n^2 + n}{2} d,$$

or

$$\sum_{i=1}^{n} x_i = \frac{n}{n-1} \cdot k + \frac{n(n-1)}{2} d.$$

Subtract each of the given equations from this equation and obtain

$$x_n = \frac{k}{n-1} + (n-1)d,$$

$$x_{n-1} = \frac{k}{n-1} + (n-2)d,$$

$$x_{n-2} = \frac{k}{n-1} + (n-3)d,$$

$$\cdots\cdots\cdots\cdots$$

$$x_2 = \frac{k}{n-1} + d,$$

$$x_1 = \frac{k}{n-1}.$$

Also solved by NATHAN ALTSHILLER, S. A. JOFFE, J. W. CLAWSON, FRANK R. MORRIS, ELBERT H. CLARKE, HORACE OLSON, N. P. PANDYA, and the PROPOSER.

431. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Form a magic square of 9 cells such that (the integers being all different) the products of the integers in the rows, columns, and diagonals shall be the same and the smallest product possible.